1. Homework Problem (Due 25 July 2011)

A football is kicked at an angle $\theta_0 = 37$ degrees. The initial speed of the ball is 20.0 m/s. First we are going to assume the ball leaves the foot at ground level. Please, answer the following:

1. Find the maximum height the ball reaches.

2. Find the time of travel before the ball hits the ground.

3. Find how far away the ball hits the ground.

4. Find the velocity and acceleration vectors at the maximum height. Give the magnitude and direction.

5. Find the velocity vector just as it hits the ground. Give the magnitude and direction.

6. If instead the ball is kicked from a height of 1 meter, how far away does it hit the ground. How does this compare with the result from question 3?

Solution

It is always good to find the components of the initial velocity. With a magnitude of 20.0 m/s and an angle of 37 degrees, the components are:

$$v_{0x} = |v_0| \cos(\theta_0) = 20 \cos(37^\circ) = 15.97 \text{ m/s} \quad (1.1)$$

$$v_{0y} = |v_0| \sin(\theta_0) = 20 \sin(37^\circ) = 12.04 \text{ m/s} \quad (1.2)$$

We can describe the motion of the ball with projectile motion. The horizontal position, velocity and acceleration are

$$x(t) = v_{0x}t \quad v_x(t) = v_{0x} \quad a_x(t) = 0 \quad (1.3)$$

and for the vertical dimension we have

$$y(t) = v_{0y}t - \frac{1}{2}gt^2 \quad v_y(t) = v_{0y} - gt \quad a_y = -g \quad (1.4)$$
**Question 1**

The maximum height $y_{\text{max}}$ is reached when $t = t_{\text{max}}$. As we discussed in class, this is the moment in time where the vertical velocity changes sign. These two conditions lead to two equations

$$0 = v_{0y} - gt_{\text{max}} \quad y_{\text{max}} = v_{0y}t_{\text{max}} - \frac{1}{2}gt_{\text{max}}^2$$  \hspace{1cm} (1.5)

We use the first equation to solve for $t_{\text{max}}$:

$$t_{\text{max}} = \frac{v_{0y}}{g} = 1.23 \text{ seconds}$$  \hspace{1cm} (1.6)

then

$$y_{\text{max}} = \frac{v_{0y}^2}{2g} = 7.40 \text{ meters}$$  \hspace{1cm} (1.7)

**Question 2**

The ball will hit the ground at $t = t_g$. At this time the vertical position is $y(t_g) = 0$. This condition leads to

$$0 = v_{0y}t_g - \frac{1}{2}gt_g^2 = t_g \left( v_{0y} - \frac{1}{2}gt_g \right)$$  \hspace{1cm} (1.8)

This is equation is a second order polynomial in time and hence it has two possible solutions. The first solution is nothing new: $t_g = 0$ is when the ball was kicked and we know it was kicked from ground level. The second solution gives

$$t_g = \frac{2v_{0y}}{g} = 2.46 \text{ seconds}$$  \hspace{1cm} (1.9)

**Question 3**

To find the total horizontal distance traveled, we evaluate $x(t)$ when $t = t_g$. This gives:

$$x(t_g) = v_{0x}t_g = \frac{2v_{0x}v_{0y}}{g} = 39.24 \text{ meters} = 43.02 \text{ yards}$$  \hspace{1cm} (1.10)

**Question 4**

At the maximum height, we know that the vertical velocity vanishes. We also known that there is no horizontal acceleration. So the horizontal velocity does not change from its initial value. Hence, at the maximum height the velocity vector is:

$$\vec{v}(t_{\text{max}}) = (v_{0x}, 0)$$  \hspace{1cm} (1.11)
The magnitude is the same as \( v_{0x} \). Since the vertical component is zero, the velocity is completely horizontal in the positive direction. That is, \( \theta = 0 \) degrees.

The acceleration vector remains constant during the entire problem. It’s magnitude is \( g \) and its direction is downward.

**Question 5**

We found that when \( t = t_g = 2.46 \) seconds the football hits the ground. At this time the vertical velocity is

\[
v_y(t_g) = v_{0y} - gt_g = 12.04 - 9.80(2.46) = -12.07 \text{ m/s}
\]

The vertical velocity is negative because the ball is moving downwards. (Actually, it should have the same magnitude as \( v_{0y} \) but negative; the discrepancy is due to truncation of numbers.) As we discussed in the previous problem, the horizontal component of the velocity does not change with time. The magnitude of the velocity vector is:

\[
|v(t_g)| = \sqrt{v_{0x}^2 + v_y^2(t_g)} = 20.0 \text{ m/s}
\]

The magnitude of the velocity is the same as the initial value. The only difference now is that the vertical component is negative. So to find the direction of the final velocity we just substrat 90 degrees from the initial velocity:

\[
\theta_g = 37 - 90 = -53 \text{ degrees}
\]

**Question 6**

Now we assume that the initial height of the football is 1 meter. This only changes the equation for the vertical position:

\[
y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2
\]

The ball will hit the ground at a different time: \( t = T \). At this time we have:

\[
X_{max} = v_{0x}T \quad 0 = y_0 + v_{0y}T - \frac{1}{2}gT^2
\]

The vertical equation is a quadratic equation in time:

\[
-4.9T^2 + 12.04T + 1 = 0
\]

This has two solutions:

\[
T = -0.08 \text{ secs} \quad T = 2.53 \text{ secs}
\]
The first solution is not physical. Evaluating the horizontal position at the second solution gives:

\[ X_{max} = 40.40 \text{ meters} = 44.18 \text{ yards} \]  

The difference between kicking the ball from the ground level and kicking it from a height of 1 meter is

\[ |x_{max} - X_{max}| = 1.16 \text{ yards} \]
2. Homework Problem (Due 25 July 2011)

Four forces act on an object. All of these are given in units of newtons. Two of these forces are given in terms of their components:

\[ \vec{F}_1 = (2, 5) \quad \vec{F}_2 = (-3, 4) \]

The other forces are given in terms of their magnitudes and directions:

\[ |F_3| = 4 \text{ N} \quad \theta_3 = 35 \text{ deg} \]
\[ |F_4| = 7 \text{ N} \quad \theta_4 = 190 \text{ deg} \]

Please, answer the following:

1. Find the components of \( \vec{F}_3 \) and \( \vec{F}_4 \).
2. Draw the force diagram.
3. Find the magnitude and direction of the net force. Draw \( \vec{F}_{net} \) in your force diagram.
4. If you add a fifth force \( \vec{F}_5 \) such that the net force is zero, what is the magnitude and direction of this fifth force? Draw \( \vec{F}_5 \) in your force diagram.

Solution

Question 1

The components of \( \vec{F}_3 \) are

\[ F_{3x} = 4 \cos(35^\circ) = 3.27 \text{ N} \quad F_{3y} = 4 \sin(35^\circ) = 2.29 \text{ N} \quad \text{(2.1)} \]

Similarly for \( \vec{F}_4 \):

\[ F_{4x} = 7 \cos(190^\circ) = -6.89 \text{ N} \quad F_{4y} = 7 \sin(190^\circ) = -1.22 \text{ N} \quad \text{(2.2)} \]

Question 2

Question 3

The net force is found by adding all the force vectors:

\[ \vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \quad \text{(2.3)} \]
The horizontal component of the net force is the sum of all the horizontal components:

\[ F_{x,\text{net}} = 2 - 3 + 3.27 - 6.89 = -4.62 \, \text{N} \]  
(2.4)

Similarly for the vertical component:

\[ F_{y,\text{net}} = 5 + 4 + 2.29 - 1.22 = 10.07 \, \text{N} \]  
(2.5)

The magnitude of the net force is

\[ |F_{\text{net}}| = \sqrt{(-4.62)^2 + (10.07)^2} = 11.07 \, \text{N} \]  
(2.6)

The direction of the net force is found using

\[ \theta_{\text{net}} = \arctan \left( \frac{F_{y,\text{net}}}{F_{x,\text{net}}} \right) \approx 114.65 \, \text{degrees} \]  
(2.7)

**Question 4**

If we want to have zero net force, we can add a fifth force \( \vec{F}_5 \). This force must have the same magnitude as the net force found in the previous question, but its direction must be opposite. So the fifth force is:

\[ \vec{F}_5 = -\vec{F}_{\text{net}} \]  
(2.8)